CHRISTIAN EMINENT COLLEGE, INDORE

(Academy of Management, Professional Education and Research)

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E-Content

On

"Lattice"

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Lattice

- A Lattice is a partially ordered set where every pair of elements has both supremum and infimum.

Definition of lattice :

- A Lattice is a partially ordered set $(L,\leq)$ in which every subset consisting of two elements has a least upper bound and a greatest lower bound.

- We denote LUB $\{a,b\}$ by $a \vee b$ and call it join or sum of a and b.

- Similarly we denote GLB $\{a,b\}$ by $a \wedge b$ and call it meet or product of a and b.Lattice is a mathematical structure with two binary operations meet and join.

Examples of Lattice

- Let A be nonempty set P(A) be its power set , The partially order set

$$(P(A) , \subseteq)$$

- Is a lattice in which meet and join are the same as the operations

$\cup$ *and* $\cap$ *respectively*.

- If A has a single element say a then

$$P(A) = \{ \emptyset , \{a\}\}$$

$$\text{and } LUB\ \{ \emptyset , \{a\}\} = \{a\}, GUB\ \{ \emptyset , \{a\}\} = \emptyset$$

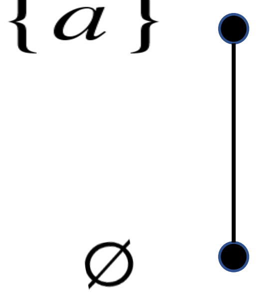
Examples of Lattice

- If A has two element say a and b

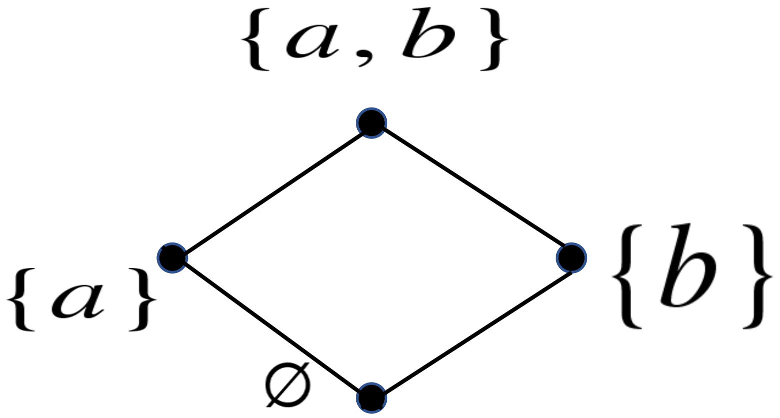
$$P(A) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$$

Hasse Diagram

- Hasse diagram for single element



- Hasse diagram for two elements



Properties of Lattices

Let $(L, \leq)$ be a lattice and let $a, b, c \in L$. Then, from the definition of $\vee$ (join) and $\wedge$ (meet) we have

(i) $a \leq a \vee b$ and $b \leq a \vee b$; $a \vee b$ is an upper bound of a and b.

(ii) if $a \leq c$ and $b \leq c$, then $a \vee b \leq c$; $a \vee b$ is the least upper bound of a and b.

(iii) $a \wedge b \leq a$ and $a \wedge b \leq b$; $a \wedge b$ is a lower bound of a and b.

(iv) If $c \leq a$ and $c \leq b$, then $c \leq a \wedge b$; $a \wedge b$ is the greatest lower bound of a and b.

Lattices as Algebraic System

Definition. A **Lattice** is an algebraic system $(L, \vee, \wedge)$ with two binary operations $\vee$ and $\wedge$, called **join** and **meet** respectively, on a non-empty set L which satisfy the following axioms for $a, b, c \in L$:

L_1 : **Commutative property**

$$a \vee b = b \vee a$$

$$a \wedge b = b \wedge a$$

L_2 : **Associative property**

$$a \wedge (b \wedge c) = (a \wedge b) \wedge c$$

$$a \vee (b \vee c) = (a \vee b) \vee c$$

Lattices as Algebraic System

L_3 : **Absorption property**

$$a \wedge (b \vee c) = a$$

$$a \vee (b \wedge c) = a$$

L_4 : **Idempotent property**

$$a \vee a = a$$

$$a \wedge a = a$$