Definition of Semigroup :

Let S be a nonempty set and o be a binary operation on S. The algebraic system (s, o) is called a semigroup if the operation is associative.

In other words, (s, o) is a semigroup if for any $x, y, z \in S$

$$(x o y) o z = x o (y o z)$$

Definition of Monoid :

Let M be a nonempty set and o be a binary operation on M. The algebraic system (M, o) is called a monoid if for any $x, y, z \in M$

$$(x o y) o z = x o (y o z)$$

And there exists an element $e \in M$ such that for any $x \in M$

$$e \; o x = x o \; e = x.$$

Example:

1. $(N,+)$ is a semigroup, where N be the set of natural numbers.

2. $(N,\times)$ is a semigroup, where N be the set of natural numbers.

3. $(N,-)$ is not a semigroup, where Z be the set of integers numbers.

3. Let S be a nonempty set and $\rho(S)$ be its power set .The algebra $(\rho(S),\cap)$ is a semigroup.

4. Let S be a nonempty set and $\rho(S)$ be its power set .The algebra $(\rho(S),\cup)$ is a semigroup.

5. $(N,+)$ is a monoid with identities element 0, where N be the set of natural numbers.

6. $(N,\times)$ is a semigroup with identities element 1, where N be the set of natural numbers.

7. Let S be a nonempty set and $\rho(S)$ be its power set .The algebra $(\rho(S),\cap)$ is a monoid with identity element S.

8. Let S be a nonempty set and $\rho(S)$ be its power set .The algebra $(\rho(S),\cup)$ is a monoid with identity element ϕ .

Homomorphism of Semigroup:

Let $(s,*)$ and (T,Δ) be any two semigroups. A mapping $g : S \to T$ such that for any two elements

$a,b \in S, \quad g(a*b) = g(a)\ \Delta\ g(b)$ is called a semigroug homomorphism.

Example:

Let Z be the set of integers and E be the set of all even integers. $(Z,+)$ and $(T,+)$ are semigroups.

Mapping $f : Z \to T$ defined by $f(a) = 2a, a \in Z$ is a semigroup homomorphism because

$f(a+b) = 2(a+b) = 2a + 2b = f(a) + f(b).$

Homomorphism of Monoid:

Let $(s,*,e_M)$ and (T,Δ,e_T) be any two monoids. A mapping $g : M \to T$ such that for any two elements

$a,b \in M, \quad g(a*b) = g(a)\ \Delta\ g(b) and\ g(e_M) = e_T$ is called a monoid homomorphism.

Example:

Let Z be the set of integers and E be the set of all even integers. $(Z,+)$ and $(T,+)$ are monoids.

Mapping $f : Z \to T$ defined by $f(a) = 2a, a \in Z$ is a monoid homomorphism because

$f(a+b) = 2(a+b) = 2a + 2b = f(a) + f(b).$